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Dissipative phase transition in two-dimensional d-wave Josephson junctions

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Abstract

Quantum dynamics of in-plane Josephson junctions between two d-wave superconducting films is described by the anisotropic XY-model, where both quasiparticle and Cooper pair tunnelling terms appear to be equally non-local. Applying a combination of the weak and strong coupling analyses to this model, we find compelling evidence of a dissipative phase transition. The corresponding critical behaviour is studied and contrasted with that found previously in the conventional (s-wave) Josephson junctions.

Quantum phenomena in ultrasmall Josephson junctions between close pairs of superconductors have long been at the forefront of research in theoretical and experimental condensed matter physics [1]. However, putting a few exceptions aside, the previous theoretical work was almost exclusively focused on the Josephson junctions between conventional s-wave superconductors which are fully gapped.

The growing interest in unconventional (e.g. d-wave) Josephson junctions has been bolstered by the studies of cuprates and other gapless superconductors [2, 3] as well as the proposed applications of mesoscopic d-wave devices in quantum computing [4].

A unifying theoretical description of tunnel junctions between both, normal metals and superconductors, can be constructed in terms of the imaginary-time effective action which governs the dynamics of the phase difference across the junction $\phi(\tau) = \phi_R(\tau) - \phi_L(\tau)$ conjugate to the junction's charge $q(\tau)$ [1]. Under the assumption of weak tunnelling, this action can be written in the general form

$$S = \frac{1}{4E_c}\int_0^{1/T}\left(\frac{\partial\phi}{\partial\tau}\right)^2 d\tau - \int_0^{1/T}\int_0^{1/T}\left[\alpha(\tau_1-\tau_2)\cos\frac{\phi(\tau_1)-\phi(\tau_2)}{2} + \beta(\tau_1-\tau_2)\cos\frac{\phi(\tau_1)+\phi(\tau_2)}{2}\right] d\tau_1\, d\tau_2 \quad (1)$$

where T is the temperature and the first (local) term is the charging energy (measured in units of $E_c = e^2/2C$) of a junction with capacitance C, while the (potentially non-local) α- and

β-terms represent the processes of quasiparticle and Cooper pair tunnelling across the junction, respectively. The corresponding integral kernels

$$\alpha(\tau) = -\sum_{k,k'} |t_{k,k'}|^2 G_{\rm L}(\tau, k) G_{\rm R}(-\tau, k')$$
$$\beta(\tau) = \sum_{k,k'} |t_{k,k'}|^2 F_{\rm L}(\tau, k) F_{\rm R}(-\tau, k') \qquad (2)$$

are determined by the Fourier transforms of the normal and anomalous quasiparticle Green functions on the left/right bank of the junction ($G_{\rm L,R}({\rm i}\omega_n, k) = ({\rm i}\omega_n + \xi_{\rm L,R})/(\omega_n^2 + \xi_{\rm L,R}^2 + \Delta_{\rm L,R}^2)$ and $F_{\rm L,R}({\rm i}\omega_n, k) = \Delta_{\rm L,R}/(\omega_n^2 + \xi_{\rm L,R}^2 + \Delta_{\rm L,R}^2)$, respectively) and the tunnelling matrix element $t_{k,k'}$.

Under the standard assumption of a generic, non-momentum-conserving, tunnelling ($t_{k,k'} \approx$ const) between two fully gapped (s-wave) superconductors, the pair tunnelling kernel decays exponentially ($\beta_s(\tau) \propto \exp(-2\Delta_s\tau)$ for $\tau \gg 1/\Delta_s$). As a consequence, the constituents of a Cooper pair tunnel almost simultaneously, and the β-term reduces to the single time integral $-E_{\rm J}\int_0^{1/T} \cos\phi(\tau)\,{\rm d}\tau$ proportional to the *local* Josephson energy $E_{\rm J} = \int_0^{1/T} \beta(\tau)\,{\rm d}\tau$.

Although the same argument may seem to be equally applicable to the quasiparticle tunnelling (α-) term, a proper description of the experimental data on realistic junctions requires one to retain a non-local quasiparticle tunnelling term whose kernel behaves as $\alpha_s(\tau) \propto 1/\tau^2$ in the entire range $1/\Delta_s < \tau < 1/T$. In this way, one can account for a finite subgap resistance due to inelastic (phase-breaking) scattering [1].

In contrast, a derivation of the effective action (1) of a junction between two gapless d-wave (as well as any $l \neq 0$-wave) superconductors along the same lines manifests its non-universality and a strong dependence on the details of the tunnelling matrix elements. For a momentum-independent tunnelling the kernel $\beta(\tau)$ given by equation (2) vanishes identically, alongside the angular averages performed in the course of the momentum integrations $\sum_k F_{\rm L,R}(\tau, k)$.

By contrast, in the presence of a momentum-conserving ($|t_{k,k'}|^2 \sim \delta^2(k - k')$) node-to-node tunnelling across the junction between two three-dimensional d-wave superconductors both kernels in equation (1) demonstrate the algebraic decay $\alpha_{d,3{\rm D}} \sim \beta_{d,3{\rm D}} \propto 1/\tau^3$, thereby resulting in the super-Ohmic quasiparticle dissipative term in equation (1) [2, 3]. Moreover, the Ohmic quasiparticle tunnelling term might still occur for those relative orientations between the d-wave order parameters that allow nodal quasiparticles to tunnel directly into the surface-bound zero energy states [3, 4].

Interestingly enough, in spite of its being of an equally non-local nature, the kernel of the concomitant Cooper tunnelling term is routinely treated as though it was local, and it has become customary to endow the effective action (1) with the ad hoc conventional Josephson energy [2–4].

The latter approximation may indeed prove adequate for the junctions formed by three-dimensional grains of a d-wave superconductor, as, e.g., in the case of strongly inhomogeneous high-$T_{\rm c}$ samples [5]. However, it would obviously fail in the case of tunnelling between a pair of genuinely two-dimensional d-wave superconductors (e.g., thin films) where both the quasiparticle and Cooper kernels show the same Ohmic decay [6]

$$\alpha_{d,2{\rm D}} = \alpha/\tau^2, \qquad \beta_{d,2{\rm D}} = \beta/\tau^2. \qquad (3)$$

In the presence of elastic scattering, this Ohmic behaviour of *both* $\alpha(\tau)$ and $\beta(\tau)$ (see erratum in [6]) will undergo a crossover to the exponential decay at timescales in excess of the inverse bulk impurity scattering rate γ, thus effectively restoring the local Josephson energy. However, as suggested by the wealth of transport data in quasi-planar cuprates, γ turns out to be quite low compared to the maximum gap Δ_d, and, therefore, the kernels (3) remain essentially non-local in the entire regime $\max[1/\Delta_d, 1/E_c] < \tau < \min[1/T, 1/\gamma]$.

Apart from its anticipated relevance to the problem of tunnelling between thin d-wave films, the anisotropic XY-model described by the effective action (1), (3) is interesting in its own right, for it appears to reside outside the realm discussed in the literature to date and its quantum dynamics has not yet been studied. To fill in the gap, in what follows we investigate this new model by applying a number of complementary techniques that cover both weak and strong dissipative coupling regimes.

In the weak coupling regime ($\alpha, \beta < 1$), an adequate approach to the model (1), (3) is provided by a direct perturbative expansion for the grand partition function in the charge representation

$$\begin{aligned}\mathcal{Z}(Q) = \sum_{n=-\infty}^{\infty} \int_{Q+ne}^{Q+ne} Dq(\tau) \sum_{N=1}^{\infty} \frac{1}{N!} \prod_{i=1}^{N} \int_0^{1/T} \mathrm{d}\tau_i^+ \int_0^{1/T} \mathrm{d}\tau_i^- \alpha(\tau_i^+ - \tau_i^-) \\ \times \sum_{M=1}^{\infty} \frac{1}{M!} \sum_{q_j=\pm 1, \sum q_j = 0} \prod_{j=1}^{M} \int_0^{1/T} \mathrm{d}\tau_j^+ \int_0^{1/T} \mathrm{d}\tau_j^- \beta(\tau_j^+ - \tau_j^-) \\ \times \exp\left(-\frac{1}{4E_c} \int_0^{1/T} q^2(\tau)\, \mathrm{d}\tau\right)\end{aligned} \tag{4}$$

where the lower limit in all the integrals is set at $\tau_c \sim 1/E_c$, and the instantaneous value of the total charge of the junction

$$q(\tau) = Q + ne + e\sum_{i=1}^{N} (\theta(\tau - \tau_i^+) - \theta(\tau - \tau_i^-)) + e \sum_{j=1}^{M} q_j[\theta(\tau - \tau_j^+) + \theta(\tau - \tau_j^-)] \tag{5}$$

includes a continuously varying contribution $Q = CV$ induced by an applied external bias V.

The sum in (4) accounts for all the trajectories in the charge space ($Q + ne \to Q + (n \pm 1)e \to \cdots \to Q + ne$) that consist of N pairs of quasiparticle ($Q \to Q \pm e$ followed by $Q' \to Q' \mp e$) and M pairs of Cooper pair ($Q \to Q \pm e$ followed by $Q' \to Q' \pm e$) tunnelling events.

The periodic dependence of the partition function (4) (hence, any physical observable) upon the external charge Q with the period e allows one to restrict its values to the 'Brillouin zone' (BZ) $-e/2 \leqslant Q \leqslant e/2$ in the charge space. Unlike in the case of the local Josephson energy, there is no room for a periodicity with the minimum charge $2e$ even for $\alpha \to 0$.

In the absence of tunnelling, the ground and first excited states appear to be degenerate ($Q^2/2C = (Q \pm e)^2/2C$) at the BZ boundaries ($Q = \pm e/2$). In the vicinity of the degeneracy points the renormalized gap $\Delta(Q) = E_1(Q) - E_0(Q)$ between the ground and first excited states remains small compared to E_c, and, therefore, one can neglect any transitions between these two energy levels and the rest of the spectrum separated by an energy gap of order E_c. As a result, the sum (4) reduces to that over the trajectories comprised of a sequence of $\Delta Q = \pm e$ 'blips' between the two lowest states [7].

A straightforward analysis shows that this effective two-state system can be amenable to the renormalization group (RG) analysis, akin to that developed in the context of the Kondo and other quantum spin-1/2 problems. In particular, the standard procedure of changing the cut-off in the time integrations in (4) from τ_c to $\tau_c' > \tau_c$ and integrating over all the pairs of opposite blips with separations $\tau_c < |\tau_i^+ - \tau_j^-| < \tau_c'$ results in the renormalized partition function whose relevant part retains the original form (4), albeit with renormalized parameter values.

To first order in the dissipative couplings, the wavefunction renormalization Z (defined through the propagators of the two lowest states, $G_{0,1} = Z\mathrm{e}^{-E_{0,1}\tau}$) and the vertex

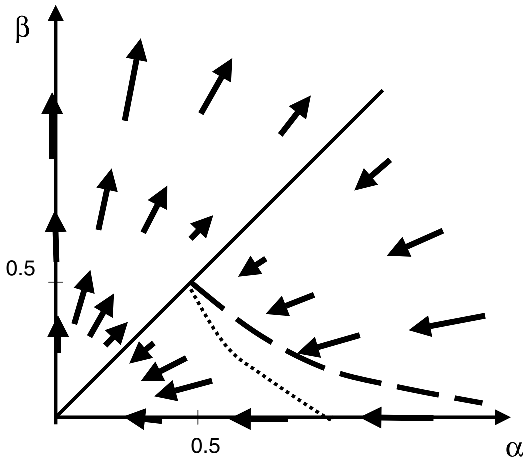

Figure 1. Phase diagram of the anisotropic XY-model. Renormalization group trajectories are shown by arrows, and the dashed (dotted) line represents the phase boundary between insulating and (super)conducting phases in the case of Ohmic (sub-Ohmic) dissipation.

renormalization factor Z_Γ receive contributions of order $\alpha \ln \tau_c'/\tau_c$ and $\beta \ln \tau_c'/\tau_c$, respectively (notably, the closed Dyson-type equations for the propagators $G_{0,1}(\tau)$ and the vertex function Γ contain no analogue of the polarization function [7]).

Combining these results, one arrives at the RG equations for the effective couplings $\alpha_r = \alpha Z^2 Z_\Gamma^2$, $\beta_r = \beta Z^2 Z_\Gamma^2$, and $\tilde{\Delta}_r = \Delta_r \tau_c$

$$\begin{aligned} \frac{d\alpha_r}{d \ln \tau_c} &= -2\alpha_r(\alpha_r - \beta_r) \\ \frac{d\beta_r}{d \ln \tau_c} &= -2\beta_r(\alpha_r - \beta_r) \\ \frac{d \ln \tilde{\Delta}_r}{d \ln \tau_c} &= (1 - 2\alpha_r) \end{aligned} \tag{6}$$

which are valid for as long as the renormalized dissipative couplings α_r and β_r remain small.

By solving the RG equations (6) and evaluating their solutions at the lowered energy cut-off $1/\tau_c' = \Delta_r$ one obtains the renormalized couplings

$$\alpha_r = \frac{\alpha\beta_r}{\beta} = \frac{\alpha}{1 + 2\eta \ln E_c/\Delta_r} \tag{7}$$

where $\eta = \alpha - \beta$. In addition, the RG procedure yields a self-consistent equation for the renormalized gap

$$\Delta_r = \frac{\Delta_0}{[1 + 2\eta \ln E_c/\Delta_r]^{\alpha/\eta}} \tag{8}$$

the bare value of which is $\Delta_0(Q) = E_1^{(0)}(Q) - E_0^{(0)}(Q) = E_c(1 - 2Q/e)$.

As follows from (7), for $\eta > 0$ the RG trajectory flows towards weak coupling (see figure 1) where the invariant charge $\tilde{\Delta}_r$ increases, although the actual gap Δ_r given by (8) continues to decrease. In this regime, the quantum current fluctuations associated with the phenomenon of Coulomb blockade completely destroy the classical Josephson effect, and the junction remains in the insulating state. This behaviour appears to be similar to that emerging in the previously studied limit $\beta = 0$ where the system possesses the exact XY-symmetry and remains insulating for arbitrary values of the external charge Q [7].

Near the line $\eta = 0$ the RG flow slows down, and right on that line the effective coupling $\alpha_r = \beta_r$ undergoes (almost) no renormalization, while the effective splitting demonstrates a power-law dependence on its bare value

$$\Delta_r = \Delta_0(\Delta_0/E_c)^{2\alpha/1-2\alpha}, \tag{9}$$

which behaviour is indicative of a possible dissipative phase transition at $\alpha = \beta = 1/2$.

Exactly at $\eta = 0$ the system acquires the Ising symmetry, which suggests that the transition in question is likely to be of the Kosterlitz–Thouless type [7]. Moreover, since the effective coupling $\alpha = \beta$ undergoes no renormalization (at least to first order), this critical behaviour appears to be reminiscent of that of a junction with the *local* Josephson energy and *Ohmic* (as opposed to the quasiparticle tunnelling-induced) dissipation.

In the regime $\alpha = \beta < 1/2$, the quantum phase fluctuations are strong and the energy bands $E_{0,1}(Q)$ remain non-degenerate, thereby forcing the junction to operate in the insulating (Coulomb-blockade-dominated) regime. However, for $\alpha = \beta > 1/2$ the quantum fluctuations get quenched, and the energy bands become progressively more and more degenerate in a finite portion of the BZ which expands from the boundaries $Q = \pm e/2$ inward as the parameter $\alpha = \beta$ increases. This behaviour indicates a suppression of the Coulomb blockade and a possible restoration of the classical Josephson effect where the voltage drop across the junction vanishes and the current is determined by a (nonzero) average value of the phase difference $\langle\phi(\tau)\rangle = \mathrm{const}$ of the superconducting order parameters on the opposite banks of the junction.

In contrast, for $\eta < 0$ one finds a runaway RG flow towards strong coupling (see figure 1) where both α_r and β_r become of order unity and $\tilde{\Delta}_r$ starts to decrease, regardless of the bare values of these dissipative couplings. Such a behaviour suggests that the junction is likely to remain in the (super)conducting regime for all $\alpha < \beta$.

Using the formula (8) for the gap, one can readily compute the ground state energy $E_0(Q) = E_c - \frac{1}{2}\Delta_r(Q)$, which, in turn, yields the junction's effective capacitance

$$C_r = \left(\frac{d^2E_0(Q)}{dQ^2}\right)^{-1}\Bigg|_{Q=0} \approx C[1 + 4\eta + O(\eta^2)]^{\alpha/\eta}. \tag{10}$$

Even more pronounced appears to be the effect of tunnelling on the average excess charge of the junction close to the BZ boundary ($\delta Q = Q - e/2 \to 0^-$)

$$\langle q(\tau)\rangle = Q - C\frac{dE_0(Q)}{dQ} = \frac{e}{2}\left(1 + \frac{\operatorname{sgn}\delta Q}{[1 + 2\eta\ln(e/\delta Q)]^{\alpha/\eta}}\right). \tag{11}$$

Consistent with the anticipated dissipation-induced suppression of the Coulomb blockade, equation (11) demonstrates the rounding of the classic Coulomb staircase-like dependence of $\langle q(\tau)\rangle$ on Q with increasing α.

Also, by continuing equation (8) analytically to the regime $Q > e/2$ where the logarithmic function of δQ acquires an imaginary part, one can estimate the decay rate of the first excited state with energy $E_1(e/2 - \delta Q) = E_0(e/2 + \delta Q)$

$$\Gamma(Q) = \operatorname{Im} E_1(Q) \approx E_c\frac{2\pi\alpha(\delta Q/e)}{[1 + 2\eta\ln(e/\delta Q)]^{1+(\alpha/\eta)}}. \tag{12}$$

The excited state remains well defined for as long as $\Gamma(Q) \ll \Delta_r(Q)$, which condition can only be satisfied in the weak coupling regime ($\eta\ln(e/\delta Q) < 1$).

Notably, the effects of the external charge screening and the reduction of the tunnelling rate manifested by equations (11) and (12), respectively, are both enhanced as compared to the case of a normal junction ('single electron box') where $\beta = 0$ [7].

These two effects are also exhibited by the current–voltage (I–V) characteristics. In a voltage-biased junction, the induced DC current

$$I(V) = \langle q(\tau)\rangle \Gamma(Q)|_{Q=VC} \approx \frac{2\pi\alpha(V - E_c/e)}{[1 + 2\eta \ln(E_c/eV - E_c)]^{1+2(\alpha/\eta)}}\theta(V - E_c/e) \tag{13}$$

vanishes below the threshold $V_c = E_c/e$. However, at finite temperatures the hard Coulomb gap is partially filled with thermally excited quasiparticle excitations, thus giving rise to a temperature-dependent conductance. By the same token, in a current-biased junction the zero-temperature I–V characteristics are expected to become non-linear for $I < eE_c$.

Next, we consider the regime of strong dissipative couplings ($\eta > 1$) where the first insight into the problem can be obtained by virtue of an adaptation of the variational technique. In this method, the correlation function of the small ('spin-wave-like') phase fluctuations is sought out in the form

$$\langle|\phi_\omega|^2\rangle = \frac{1}{\omega^2/E_c + g|\omega| + D}. \tag{14}$$

The variational parameters g and D are then used to minimize the free energy $F = -T\ln \mathcal{Z}_0 + T\langle S - \delta^2 S_0\rangle$, where $\delta^2 S_0(\phi_\omega)$ is the quadratic action which corresponds to equation (14) and determines the averages $\langle e^{i\phi}\rangle = \exp(-\frac{1}{2}\langle\phi^2\rangle)$. In this way, one obtains a self-consistent equation for the effective dissipative coupling

$$\eta = g^{(g+1/g-1)}(2\beta)^{1/1-g}. \tag{15}$$

It can be shown that if the condition

$$\alpha - \beta > e\theta(1/2 - \beta)\ln\frac{1}{2\beta} \tag{16}$$

is fulfilled (here e is the Euler's number), there exists a finite gap due to the XY-anisotropy in the two-dimensional vector space spanned by the unit vector $\mathbf{n} = (\cos\phi/2, \sin\phi/2)$

$$D = E_c(2\beta)^{g/g-1}g^{2/1-g} \tag{17}$$

where $g \approx \eta^{1-2/\eta}(2\beta)^{1/\eta}$ is the solution of equation (15) for $\eta > 0$. For comparison, within almost the entire domain $\eta < 0$ one finds $g \approx \eta$.

The gap (17) attains its bare value $2\beta E_c$ at $g \to \infty$ and decreases upon lowering the energy cut-off. Considering that the bandwidth of the phase fluctuations, too, undergoes a downward renormalization from its bare value E_c (see equation (28) below), one finds that the spectrum of the quadratic operator $\delta^2 S_0/\delta\phi^2$ remains stable for both positive and negative η.

It should also be noted that, when continued to real frequencies, the spectrum of the small phase fluctuations appears to be strongly overdamped. Therefore, except for the immediate vicinity of the separatrix $\eta = 0$ (hence, in almost the entire α–β plane), it bears no resemblance to the simple plasmon pole (cf [6]).

In the presence of a nonzero XY-anisotropy $D \neq 0$, the expectation value $\langle\phi^2(\tau)\rangle$ becomes finite and the phase ϕ is localized. As a result, the real part of the AC conductance

$$G(\omega) = I(\omega)/V(\omega) = \mathrm{Re}\left.\frac{1}{\omega_n\langle|\phi_{\omega_n}|^2\rangle}\right|_{\omega_n\to -i\omega} \tag{18}$$

develops a coherent peak at zero frequency, $G(\omega \to 0) = D\delta(\omega)$, thus indicating the onset of the classical Josephson effect.

In the case where equation (15) features no solution, the gap D vanishes and the expectation value of the phase fluctuations $\langle\phi^2(\tau)\rangle$ diverges, so that the phase remains delocalized. At first sight, it may seem that the real part of the conductance approaches a finite value $G(0) = g$ in the DC limit. However, we expect that the continuing downward renormalization of the effective dissipative parameter η (see equation (19) below) will eventually bring the system into the insulating regime.

In order to justify the above claim, we point out that in the absence of a non-trivial mean field solution the variational method must be abandoned in favour of a straightforward perturbative approach. The latter yields a continuous renormalization of the dissipative couplings due to the non-Gaussian (quartic and higher order) terms in the action (1).

To the leading order, such a renormalization is described by the RG equations

$$\frac{\mathrm{d}\alpha_{\mathrm{r}}}{\mathrm{d}\ln\tau_{\mathrm{c}}} = -\frac{\alpha_{\mathrm{r}}}{2\pi^2\eta_{\mathrm{r}}}, \qquad \frac{\mathrm{d}\beta}{\mathrm{d}\ln\tau_{\mathrm{c}}} = -\frac{\beta_{\mathrm{r}}}{2\pi^2\eta_{\mathrm{r}}} \tag{19}$$

which demonstrate a monotonic decrease of the effective dissipative parameter as a function of the lowered energy cut-off $1/\tau_{\mathrm{c}}'$

$$\eta_{\mathrm{r}} = \eta - \frac{1}{2\pi^2}\ln E_{\mathrm{c}}\tau_{\mathrm{c}}'. \tag{20}$$

It is worth noting that the steady decrease of both α_{r} and β_{r} as well as the approximate constancy of the ratio $\alpha_{\mathrm{r}}/\beta_{\mathrm{r}}$ manifested by equations (19) are consistent with the behaviour found in the weak coupling regime, and so the RG trajectories interpolate smoothly between the weak and strong coupling regimes.

Interestingly enough, the RG equations (19) coincide with those of the spin-boson model

$$S_{\mathrm{sb}}[\mathbf{m}(\tau)] = \frac{\mathrm{i}}{2}\int \mathrm{d}\tau(1-m_z)\frac{\partial\phi}{\partial\tau} + \int \mathrm{d}\tau\left(\frac{1}{4}E_{\mathrm{c}}m_z^2 + m_x h_x + m_y h_y\right) \tag{21}$$

where $\mathbf{m} = (\sqrt{1-m_z^2}\cos\phi/2, \sqrt{1-m_z^2}\sin\phi/2, m_z)$ is a 3D unit vector field composed of the phase ϕ and a conjugate 'momentum' m_z. The first term in (21) is the spin's Berry phase, and a two-component random field $h_{x,y}(\tau)$ represents a dissipative bath governed by the correlators $\langle|h_x(\omega)|^2\rangle = (\alpha+\beta)|\omega|$ and $\langle|h_y(\omega)|^2\rangle = \eta|\omega|$.

The origin of the equivalence between the strong coupling regimes of the actions (1), (3) and (21) can be traced back to the observation that at large η both the discreteness of charge and the quantum nature of spin (or, for that matter, the spin's Berry phase) turn out to be largely irrelevant and the actions (1), (3) and (21) are both dominated by the (identical) dissipative terms.

As another side note, we mention that the action (21) can also be encountered in generic (spin anisotropic) models of quantum impurities as well as those of noisy qubits (generalized (pseudo)spin-1/2 systems exposed to dissipative environments). In light of the earlier observations [9] that in such models the only stable fixed points tend to be those of the Ising symmetry, it does not seem unreasonable to surmise that the behaviour in the entire region defined by equation (16) can, in fact, be similar to that found for $\alpha = \beta > 1/2$.

Altogether, the above results suggest a tentative layout of the phase diagram where the insulating behaviour sets in throughout the domain defined by the inequalities $0 \leqslant \beta < 1/2$ and $\beta \leqslant \alpha \leqslant \beta + e\ln(1/2\beta)$ (see figure 1). By implication, we then conjecture that upon increasing the value of $\eta > 0$ or changing its sign, the insulator gives way to the (super)conducting phase.

In order to further refine our understanding of the strong coupling regime we employ the semiclassical approach in the phase representation [8]. The Coulomb energy (which, in contrast to the weak coupling case, is now subdominant to the dissipative terms) induces a splitting between the (otherwise degenerate) phase configurations ('vacua'), and the partition function

$$\mathcal{Z}(Q) = \sum_{N=0}^{\infty}\int_0^{2\pi}\mathrm{d}\phi_0\int_{\phi_0}^{\phi_0+4\pi N}\mathrm{D}\phi(\tau)\mathrm{e}^{-S[\phi(\tau)]+2\pi\mathrm{i}NQ/e} \tag{22}$$

is now saturated by the trains of 4π-phase slips ('instantons') corresponding to the transitions between different even/odd vacua $\phi_{2M} = 2\pi M$ and $\phi_{2M+1} = (2M+1)\pi$.

A viable candidate to the role of the trajectory connecting the vacua $\phi_{2M(2M+1)}$ and $\phi_{2M+N(2M+1+N)}$ can be chosen in the form (here $q_k = \pm 1$ and $\sum_{k=1}^{n} q_k = N$)

$$\Delta\phi_N(\tau) = 4\sum_{k=1}^{n} q_k \tan^{-1}\Omega_k(\tau - \tau_k) \tag{23}$$

which is inspired by the exact solution of the classical equations of motion for $\beta = 0$ and $E_c = \infty$.

Evaluating the action (1), (3) on the single instanton ('bounce') trajectory $\Delta\phi_1(\tau)$ we obtain

$$S_1(\Omega) = 2\pi^2\eta + \frac{\pi\Omega}{2E_c} + \frac{4\pi\beta E_c}{\Omega}. \tag{24}$$

Minimizing equation (24) with respect to the instanton's size Ω, we find $\Omega_{\min} \sim E_c\beta^{1/2}$, which is small compared to the average spacing between the instantons ($|\tau_n - \tau_m| \sim 1/NT$) for $T \to 0$.

Since the instantons are well separated, one can expand about a single-instanton configuration ($\phi(\tau) = \Delta\phi_1(\tau) + \delta\phi(\tau)$) and obtain a quadratic action of the Gaussian fluctuations

$$\begin{aligned}\delta^2 S_1 = T\sum_{\omega}(\alpha|\delta\phi_\omega|^2[|\omega + \Omega/2| + |\omega - \Omega/2| - |\Omega|]\\ + \beta[\delta\phi_\omega\delta\phi^*_{\Omega-\omega}(E_c/T - |\omega + \Omega/2| - \Omega/2)\\ + \delta\phi_\omega\delta\phi^*_{\Omega+\omega}(E_c/T - |\omega - \Omega/2| - \Omega/2)]).\end{aligned} \tag{25}$$

At $\beta = 0$ the eigenvalues of the operator $\delta^2 S_1$ with the frequencies $|\omega| < \Omega$ turn out to be independent of α. However, for any finite β these 'soft' modes acquire gaps and, therefore, cannot be separated from the rest of the spectrum (cf [8]). The fact that this spectrum remains positively defined and, therefore, stable justifies the use of equation (24) as a sensible approximation for the (unknown) true local minimum of the action (1), (3).

Next, by singling out the zero modes corresponding to the changes in the instanton position ($\delta\phi_1^{(1)} \propto \partial\phi_1/\partial\tau$) and size ($\delta\phi_1^{(2)} \propto \partial\phi_1/\partial\Omega$) and integrating over the non-zero normal modes, one can cast the partition function as that of a dilute instanton gas

$$\mathcal{Z}(Q) \approx \sum_{N=1}^{\infty}\frac{1}{N!}\prod_{k=1}^{N}\int_0^{1/T} d\tau_k \int d\Omega_k\, J(\Omega_k)\left(\frac{\det\delta^2 S_0/\delta\phi^2}{\det\delta^2 S_1/\delta\phi^2}\right)^{1/2} e^{-S_1(\Omega_k)+2\pi i NQ/e} \tag{26}$$

where $J(\Omega)$ is the measure of integration over the zero modes, and the determinant stemming from the integration over the non-zero modes is normalized with respect to that of the operator $\delta^2 S_0/\delta\phi^2$ corresponding to the Gaussian fluctuations about the trivial vacuum $\phi(\tau) = 0$.

At large positive values of η the ground state energy can be obtained by taking the zero-temperature limit of the logarithm of equation (26)

$$E_0(Q) = -T\ln\mathcal{Z}(Q)|_{T\to 0} = E_c\cos(2\pi Q/e)\int d\Omega\, J(\Omega)\left(\frac{\det\delta^2 S_0/\delta\phi^2}{\det\delta^2 S_1/\delta\phi^2}\right)^{1/2} e^{-S_1(\Omega)}. \tag{27}$$

At $Q = 0$ equation (27) yields a renormalized bandwidth of the phase fluctuations whose asymptotic behaviour for $\eta \gg 1$ reads

$$E_{c,r} \sim E_c\eta e^{-2\pi^2\eta}. \tag{28}$$

By analogy with the $\beta = 0$ case, this reduction of the bandwidth can be attributed solely to the exponential blow-up of the effective capacitance $C_r \sim C\exp(2\pi^2\eta)/\eta$. For comparison, along the separatrix $\alpha = \beta$, the effective capacitance grows only as $C_r \sim C\exp(\pi\sqrt{\alpha}/8)/\alpha^{1/4}$ for large α.

The advantage of working in the phase representation is that one can better elucidate the nature of the putative dissipative phase transition by relating it to a spontaneous breaking of the symmetry between the ϕ_{even} and ϕ_{odd} vacua [8].

In the disordered phase, the correlation function $\langle e^{i\phi(\tau)/2}e^{-i\phi(0)/2}\rangle$ decays algebraically and the insulator-like I–V characteristics show the presence of a hard (at $T = 0$ and $I \rightarrow 0$) Coulomb gap. By contrast, in the ordered phase the system develops an order parameter $\langle e^{i\phi(\tau)/2}\rangle \neq 0$ whose presence indicates that the phase variable is localized in either even or odd vacua. This does not, however, constitute a complete phase localization, and, therefore, the apparent classic Josephson effect should be considered a non-equilibrium phenomenon which can only be observed at finite (albeit potentially quite long) observation times, while at still longer times the response would revert to the resistive behaviour [1].

Another potentially important non-equilibrium effect is excitonic enhancement of the tunnelling probability, which modifies the exponent in the power law in equations (3) to $2 - \epsilon$, where ϵ is a function of the (non-universal) scattering phase shift [10]. In this 'sub-Ohmic' case, the right-hand sides of the RG equations (19) pick up additional terms $\epsilon\alpha$ and $\epsilon\beta$. Consequently, one now finds a fixed point at $\eta_c = 1/2\pi^2\epsilon$, upon approaching which the bandwidth vanishes as $E_c^* \sim E_c(1 - \eta/\eta_c)^{(1-\epsilon)/\epsilon}$. One would then be led to the conclusion that in the domain defined as $\eta_c \leqslant \alpha - \beta \leqslant e\ln(1/2\beta)$ the transition from the insulating to the Josephson phase is pre-empted by that into a new conducting (albeit, possibly, dissipative) state.

We anticipate that the prediction of the dissipative phase transition made in this work might be possible to test by using junctions between thin single-crystal cuprate films. For the parameter values found in [6] ($\alpha = 2\beta$ and $E_c \sim \Delta_d$) one would expect to observe the apparent superconducting response at $\alpha > \alpha_c \approx 0.85$. Considering that the typical junctions manufactured from the cuprate superconductors tend to have relatively high conductances [5], this condition can be readily met.

A still more realistic model of the cuprate films would also feature a renormalized kernel $\alpha(\tau)$ that accounts for possible zero-energy bound states (which is only absent in the case of the node-to-node tunnelling) as well as a quadratic Ohmic dissipative term representing the effect of a shunting normal resistance (metallic environment surrounding the superconducting grains). We anticipate that by including these effects in the model (1), (3) the borderline between the (super)conducting and insulating phases can be pushed even further towards lower values of the dissipative couplings.

In contrast to our findings, the naive gradient expansion employed in [6] is not capable of revealing any critical coupling α_c or the corresponding phase transition in question. In fact, a direct application of this technique to the model (1), (2) would yield an undamped correlation function $\langle|\phi_\omega|^2\rangle = (\omega^2/E_c^* + D^*)^{-1}$ with a strongly temperature-dependent gap $\Delta \sim (E_c T)^{1/2}$ throughout the entire α–β plane [6], which behaviour is starkly different from that found in this work.

Also, as far as a proper modelling of the highly inhomogeneous cuprate samples studied in [5] is concerned, the latter are likely to be best described not by the strictly two-dimensional action (1), (3) but rather as a network of three-dimensional grains whose behaviour is expected to be somewhat more conventional [1].

To conclude, in this work we carried out a comprehensive analysis of the quantum dynamics of a node-to-node Josephson junction between two planar d-wave superconductors. Our results reveal the presence of a dissipative phase transition in the α–β plane. We also discussed properties of the physical observables that are indicative of the transition in question, including the junction's energy spectrum, effective capacitance and I–V characteristics.

The corresponding critical behaviour differs, in a number of important aspects, from the previously studied cases of a superconducting junction with the local Josephson energy as well as that of a normal junction. However, in a general agreement with the earlier analyses of these systems the presence of a sufficiently strong quasiparticle *and* Cooper pair tunnelling turns out to be instrumental for observing the Josephson behaviour (albeit, possibly, not at the longest timescales).

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